Observations on bubble growths in various superheated liquids

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Experimental data are presented for the growth of vapour bubbles in various superheated liquids, such as carbon tetrachloride, benzene, ethyl alcohol, and methyl alcohol. These data are compared with the theoretical results obtained by Plesset & Zwick (1953) who derived these results by taking into account the heat diffusion across the bubble boundary. The agreement in all cases between experiment and theory is found to be good.

The growth of vapour bubbles in slightly superheated water is also presented in the form of experimental data for bubbles just beginning to grow from a point of equilibrium which is presumed to be dynamically unstable. The radii corresponding to the points of equilibrium are of the same order of magnitude as those predicted by theoretical considerations.

1. Introduction

The importance of the cavitation phenomenon in the field of hydrodynamics is well recognized by both the theoretical physicist and the design engineer. In particular, the engineer in the field of underwater ordnance is constantly confronted with problems having to do with performance degradation of propulsion systems and missiles. An intelligent application of the present knowledge in cavitating flow requires some insight into the basic mechanism of cavitation. The present paper is an attempt to add to this knowledge of the mechanism of cavitation rather than to furnish some tools to the design engineer for direct application to design problems.

The analogy between producing bubbles by a reduction of the external pressure and by increasing the vapour pressure of a liquid by heating is used in the present study, and hence all the experiments are conducted by superheating liquids with the use of infra-red lamps. The experimental procedures are the same as those described in a previous paper by the author (1953). In the case of carbon tetrachloride and benzene, the work was carried out in a ventilated hood. Since carbon tetrachloride is a very poor absorber of infra-red radiation, the container for the carbon tetrachloride was placed inside a larger beaker containing water. Thus the infra-red lamps heated the water which in turn increased the temperature of the carbon tetrachloride. By the use of Eastman Tri-X film, it was also possible to obtain larger magnifications with smaller lens openings. This film, together with

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the Edgerton lamps, made it possible to obtain useful data for bubble radii of the order of 10^{-3} cm.

The experimental part of this paper is an analysis of high-speed photographs of the growth of vapour bubbles in water, carbon tetrachloride, benzene, ethyl alcohol, and methyl alcohol, at superheat temperatures, in order to obtain a detailed check with the Plesset–Zwick (1953) theory for bubble growth. Previous work by the author (1953) had shown that the experimental data obtained for the growth of bubbles in superheated water indicated good agreement with theory. All of the data in this previous work were obtained in an observable range of radii considerably larger than the equilibrium bubble size predicted by the theory in the report by the author (1953). Thus, in the case of water, the present experiments are an attempt to measure bubble growth from the equilibrium point, whereas the data for the other liquids are intended to show a comparison with water for rate of growth using the same amount of superheat in each case.

The theoretical part of this paper is a detailed presentation of the analysis for calculating the range of bubble radii that are unstable and those that are stable for any given set of conditions.

2. Growth rates for various liquids

The Plesset–Zwick (1953) theory, which includes the effect of heat diffusion across the bubble wall, predicts a growth rate which has the following behaviour:

$$R \sim 2\frac{\rho c}{\rho' L}\sqrt{\left(\frac{3}{\pi}Dt\right)}\,(T-T_0) \quad (t \to \infty), \tag{1}$$

where $T-T_0$ indicates the amount of superheat in °C, ρ and ρ' are the liquid and vapour densities, respectively, c is the specific heat for the liquid, L is the latent heat of vaporization, and D is the thermal diffusivity for the superheated liquid. This formula for R, the radius as a function of the time t, indicates that, for a given amount of superheat $T-T_0$, together with vapour-pressure curves having similar slopes, the growth of bubbles in different liquids should be proportional to the quantity

$$\frac{\rho c}{\rho' L}\sqrt{D}. \tag{2}$$

For the liquids used, the values of this parameter and the variations of pressure with temperature near the pressure of 760 mm of mercury are shown in the following table:

	$\frac{\rho c}{\rho' L}\sqrt{D}$ (cm/°C sec$^{\frac{1}{2}}$)	Temp. (°C)	Variation (°C/cm)
water	0·121	100·0	0·37
carbon tet.	0·034	76·7	0·41
benzene	0·046	80·1	0·43
methyl alc.	0·046	64·6	0·35
ethyl alc.	0·044	78·3	0·34

Since the pressure variations with temperature are nearly the same, the parameter (2) predicts a growth rate for bubbles in water of approximately 3·5 times that for carbon tetrachloride, 2·6 times that for benzene and methyl alcohol, and 2·8 times that for ethyl alcohol. All of the liquids used in the experiment were chemically of reagent grade. Figure 1 shows a comparison of the growth rates. The amounts of superheat are: 5·3 °C for water, 6·3 °C for ethyl alcohol, 5·4 °C for methyl alcohol, 5·4 °C for benzene, and 5·4 °C for carbon tetrachloride. The Plesset–Zwick (1953) theory shows good agreement with the data for water, and the agreement is seen to be good for the other liquids, which all show a smaller rate

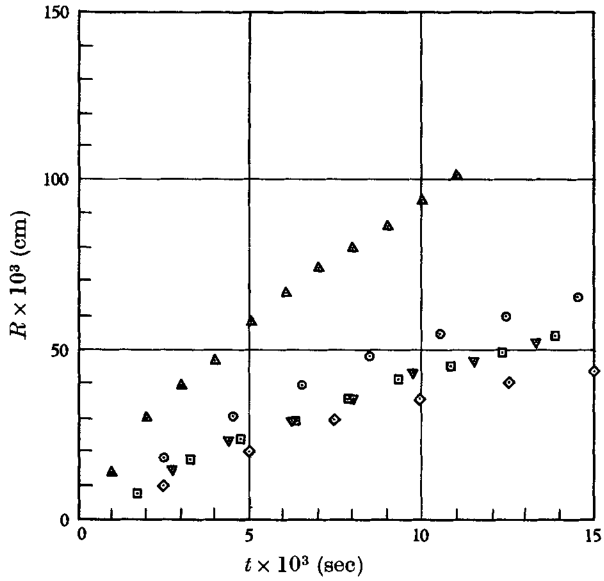

FIGURE 1. Radius as a function of time in various superheated liquids. R is shown as a function of t for vapour bubbles growing in superheated water, ethyl alcohol, methyl alcohol, benzene, and carbon tetrachloride. △, H_2O, 105·3 °C, B.P. (760 mm), 100 °C; ⊙, CH_3CH_2OH, 84·6 °C, B.P. (760 mm), 78·3 °C; ▽, CH_3OH, 70·0 °C, B.P. (760 mm), 64·6 °C; ⊡, C_6H_6, 85·5 °C, B.P. (760 mm), 80·1 °C; ◇, CCl_4, 82·1 °C, B.P. (760 mm), 76·7 °C.

of growth as predicted by the theory. The temperature measurements are presumed to be accurate within ± 0·2 °C. In addition, the true boiling points for all the liquids with the exception of water are not known within ± 0·1 °C. Because of these inaccuracies, the comparison with theory is of a qualitative nature. However, the trend is seen to be the one predicted by the theory in every case.

3. Growth rate near equilibrium

All of the data used in the previous discussion were obtained in an observable range of radii considerably larger than the equilibrium bubble size predicted by theory (see Appendix). The mechanism for bubble growth, as described in the Appendix, is that for growth from a radius corresponding to a dynamically unstable equilibrium. The subsequent growth from this equilibrium is then predicted by the Plesset–Zwick theory. To clarify this model, consider the equation of motion for the bubble

$$R\ddot{R}+\tfrac{3}{2}\dot{R}^2 = \frac{p_v - p_\infty + p_a - (2\sigma/R)}{\rho},$$

where ρ is the density of the liquid, R is the radius of the bubble at time t, p_v is the vapour pressure of the water at the appropriate temperature, p_a is the partial pressure of air which may be present in the bubble, p_∞ is the atmospheric pressure, and σ is the surface-tension constant for water. With the assumptions of the Appendix, it turns out that the range of bubble radii in terms of the amount of superheat, external pressure, and initial air content, for which a condition of dynamic instability exists is given by

$$\frac{4\sigma}{3\delta p} \leqslant R_0 \leqslant \frac{2\sigma}{\delta p}, \quad \text{with} \quad \delta p \equiv p_v - p_\infty \geqslant 2p_{a0} \geqslant 0.$$

Thus, for superheats of 3 °C and 1 °C for water,

$$R_0 \simeq 10^{-3}\,\text{cm} \quad \text{and} \quad R_0 \simeq 3\cdot 1 \times 10^{-3}\,\text{cm},$$

respectively. It is evident that to obtain data for R of the order of 10^{-3} cm, it is necessary to have relatively large magnifications. This requirement is reflected in the need for faster film and a more intense light source. With the use of Edgerton

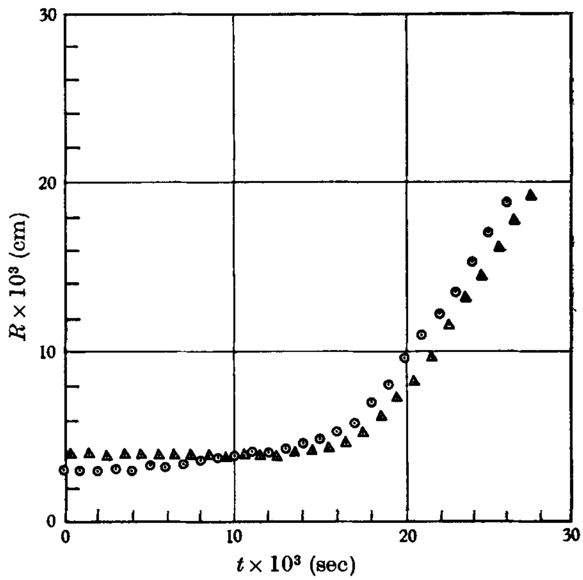

FIGURE 2. Radius as a function of time in slightly superheated water. R is shown as a function of t for two vapour bubbles growing in slightly superheated water. The growth in this case is from a radius near the equilibrium size for the given temperature. ⊙, 101·0 °C; △, 100·8 °C.

lamps and Eastman Tri-X film, the possibility of obtaining useful data for bubble radii of the order of 10^{-3} cm was investigated and found to yield satisfactory results. Figure 2 shows the results for two bubbles, one for 1 °C superheat and the other for 0·8 °C superheat. The bubbles are seen to grow very slowly from a radius between 3×10^{-3} and 4×10^{-3} cm until a point near 0·013 sec, at which time the growth rate is markedly increased. Actually, the bubbles remain nearly constant in size for a time corresponding to 0·1 sec. The value $t = 0$ was chosen arbitrarily for the graph. The bubble radii compare favourably with the predicted value of

$R_0 \simeq 3{\cdot}1 \times 10^{-3}$ cm for $T = 101\,°\mathrm{C}$. However, for these low values of superheat the growth rate is sufficiently slow for translational effects of the bubble to modify its growth rate, and a comparison with the Plesset–Zwick theory is made difficult. However, the growth from an equilibrium size seems to be indicated from the data. Higher superheats would furnish better data for a comparison with theory, but the difficulty in obtaining such data is greatly increased because of the greater growth rates and smaller initial radii involved.

Appendix

Equilibrium radii for bubble growth

(1) *Equation of motion for the growth of a cavitation bubble*

Frequent reference is made in the literature on cavitation to Rayleigh's (1917) solution for the collapse of a spherical cavity in a liquid. Rayleigh's theory can be extended to the case of growth of a bubble. Rayleigh considered the situation in which the pressure at a great distance from the bubble is constant. With this assumption, and the assumption of an incompressible fluid, the variation of the bubble radius with time is obtained from the energy integral of the motion. For the present problem of the growth of a bubble, the extension of the Rayleigh theory as carried out by Plesset (1949) can be used to obtain the equation of motion. The equation is obtained by considering a spherical bubble in a perfect incompressible fluid of infinite extent. Neglecting the effects of gravity, the origin is chosen at the bubble centre which is at rest. The radius of the bubble for any time t is R, and r is the radius to any point in the liquid. Thus the velocity potential for the liquid is expressed by

$$\phi = \frac{R^2\dot{R}}{r}, \tag{1}$$

and the Bernoulli integral of the motion is

$$-\frac{\partial\phi}{\partial t} + \tfrac{1}{2}(\nabla\phi)^2 + \frac{p(r)}{\rho} = \frac{P(t)}{\rho}, \tag{2}$$

where ρ is the constant density of the liquid, $\dot{R} = dR/dt$, $p(r)$ is the static pressure at r, and $P(t)$ is the static pressure at a large distance from the bubble. From (1),

$$(\nabla\phi)^2 = \frac{R^4\dot{R}^2}{r^4}, \tag{3}$$

$$\frac{\partial\phi}{\partial t} = \frac{1}{r}(2R\dot{R}^2 + R^2\ddot{R}), \tag{4}$$

and applying (2) at $r = R$, we then obtain the equation of motion for the bubble radius. Thus, with

$$\left(\frac{\partial\phi}{\partial t}\right)_{r=R} = 2\dot{R}^2 + R\ddot{R},$$

$$(\nabla\phi)^2_{r=R} = \dot{R}^2,$$

(2) becomes

$$R\ddot{R} + \tfrac{3}{2}\dot{R}^2 = \frac{p(R) - P(t)}{\rho}. \tag{5}$$

Equation (5) is the general equation of motion for a spherical bubble in a liquid with given external pressure $P(t)$, and with the pressure at the bubble boundary $p(R)$. Rayleigh's equation is obtained as a special case if

$$P(t)-p(R) = P_0 \text{ (a constant).}$$

Equation (5) is adapted to the present problem with the assumption that

$$p(R) = p_v + p_a - \frac{2\sigma}{R},$$

where p_v is the vapour pressure of the water at the appropriate temperature, p_a is the pressure of any air which may be in the bubble of radius R, and σ is the surface-tension constant for water. Letting

$$P(t) = p_\infty \text{ (a constant),}$$

where p_∞ is the atmospheric pressure, the equation of motion for the bubble radius becomes

$$R\ddot{R} + \tfrac{3}{2}\dot{R}^2 = \frac{p_v - p_\infty + p_a - (2\sigma/R)}{\rho}. \tag{6}$$

(2) *Solution of* (6) *with* $p_v = $ *constant*

If the assumption is made that the vapour pressure p_v remains constant throughout the growth of the bubble, then the bubble growth is isothermal, so that

$$p_a = p_{a0}\frac{R_0^3}{R^3}, \tag{7}$$

where p_{a0} is the initial pressure of the air in a bubble of radius R_0. Equation (7) implies the assumption that no air diffuses across the bubble boundary as it grows. Plesset & Epstein (1950) have shown that the diffusion process for gas bubbles is so slow compared with the rate of growth of the bubble that it does not affect the air content of the bubble. Thus (7) is a reasonable expression for the air pressure p_a as a function of the bubble radius.

From (6) it can be seen that the bubble is in dynamic equilibrium with the liquid if

$$f(R) = p_v - p_\infty + p_{a0}\frac{R_0^3}{R^3} - \frac{2\sigma}{R} = 0 \quad (\dot{R} = 0). \tag{8}$$

One obvious root of (8) is $R = R_0$, where

$$R_0 = \frac{2\sigma}{p_v + p_{a0} - p_\infty}.$$

The remaining roots of (8) are then found to be

$$R = \frac{p_{a0}\sigma}{\delta p(\delta p + p_{a0})}\left\{1 \pm \sqrt{\left(1 + 4\frac{\delta p}{p_{a0}}\right)}\right\},$$

where $\delta p = p_v - p_\infty$. If $\delta p > 0$, then the two positive roots of (8) that correspond to actual bubble radii are

$$R = \frac{2\sigma}{\delta p + p_{a0}}$$

and

$$R = \frac{p_{a0}\sigma}{\delta p(\delta p + p_{a0})}\left\{1 + \sqrt{\left(1 + 4\frac{\delta p}{p_{a0}}\right)}\right\}.$$

The case $\delta p > 0$ corresponds to the condition that the vapour pressure p_v is greater than the atmospheric pressure p_∞, so that the liquid can boil. If $\delta p \leqslant 0$, then there is only one positive root of (8), namely

$$R = \frac{2\sigma}{\delta p + p_{a0}},$$

provided that $p_{a0} > |\delta p|$.

The effect of varying the value of the initial air pressure p_{a0} on the equilibrium radii will be considered next.

For the case $\delta p > 0$, it should be noted that

$$p_{a0} R_0^3 = p_{a0}\left(\frac{2\sigma}{\delta p + p_{a0}}\right)^3$$

has the value zero for $p_{a0} = 0$, increases to a maximum value of

$$\tfrac{1}{2}\delta p(4\sigma/3\delta p)^3 \quad \text{at} \quad p_{a0} = \tfrac{1}{2}\delta p$$

and then approaches zero as $p_{a0} \to \infty$. Thus the entire range of values of $p_{a0}R_0^3$ is covered in the range $0 \leqslant p_{a0} \leqslant \frac{1}{2}\delta p$. For the case of a vapour bubble, $p_{a0} = 0$, and there is only one positive root of (8), its value being $R = 2\sigma/\delta p$. When $p_{a0} = \frac{1}{2}\delta p$, the two positive roots coincide with a value of $R = 4\sigma/3\delta p$. If any values of $p_{a0} > \frac{1}{2}\delta p$ are considered, the two roots merely interchange roles. Thus for the case $\frac{1}{2}\delta p > 0$, it is sufficient to consider the two roots

$$R = R_0 = \frac{2\sigma}{\delta p + p_{a0}}$$

for $0 \leqslant p_{a0} \leqslant \frac{1}{2}\delta p$, and

$$R = R_1 = \frac{p_{a0}\sigma}{\delta p(\delta p + p_{a0})}\left\{1 + \sqrt{\left(1 + 4\frac{\delta p}{p_{a0}}\right)}\right\}$$

$$= \frac{2\sigma}{\delta p + p_{a1}},$$

$$p_{a1} = \frac{2\delta p(\delta p + p_{a0})}{p_{a0}\left\{1 + \sqrt{\left(1 + 4\frac{\delta p}{p_{a0}}\right)}\right\}} - \delta p$$

for $p_{a1} > \frac{1}{2}\delta p$. Thus the entire range of possible values for the initial air pressure in the bubble is covered. For the case $\delta p \leqslant 0$, the only root is

$$R = R_0 = \frac{2\sigma}{\delta p + p_{a0}}$$

with $p_{a0} > |\delta p|$.

To determine whether the equilibrium of a bubble is dynamically stable or unstable, one may consider

$$\frac{df(R)}{dR} = \frac{2\sigma}{R^2} - 3p_{a0}\frac{R_0^3}{R^3} \tag{9}$$

for the given radius. Thus for dynamic stability $df(R)/dR < 0$, and for dynamic instability $df(R)/dR > 0$. For the case $\delta p > 0$ and radius

$$R = R_0 = \frac{2\sigma}{\delta p + p_{a0}},$$

(9) becomes
$$\left[\frac{df(R)}{dR}\right]_{R=R_0} = \frac{\delta p + p_{a0}}{2\sigma}(\delta p - 2p_{a0})$$

or
$$\left[\frac{df(R)}{dR}\right]_{R=R_0} \geqslant 0,$$

if $0 \leqslant p_{a0} \leqslant \frac{1}{2}\delta p$. This means that the equilibrium of a bubble of radius $R_0 = 2\sigma/\delta p + p_{a0}$, with $\delta p > 0$, is dynamically unstable if $0 \leqslant p_{a0} < \frac{1}{2}\delta p$. For the case $p_{a0} = \frac{1}{2}\delta p$, $df(R)/dR = 0$, the bubble is dynamically unstable if the radius is increased beyond R_0, but is dynamically stable if the radius is decreased below R_0. In terms of the growth of the bubble, the range of equilibrium radii are

$$\frac{4\sigma}{3\delta p} \leqslant R_0 \leqslant \frac{2\sigma}{\delta p} \quad \text{with} \quad \tfrac{1}{2}\delta p \geqslant p_{a0} \geqslant 0.$$

For the radius

$$R = R_1 = \frac{2\sigma}{\delta p + p_{a1}} \quad (p_{a1} > \tfrac{1}{2}\delta p),$$

(9) becomes
$$\left[\frac{df(R)}{dR}\right]_{R=R_1} = -3p_{a0}\frac{R_0^3}{R_1^4} + \frac{2\sigma}{R_1^2}.$$

Since $R_1 < R_0$ and $\left[\frac{df(R)}{dR}\right]_{R=R_0} = 0$ for $p_{a0} = \frac{1}{2}\delta p$,

one obtains
$$\left[\frac{df(R)}{dR}\right]_{R=R_1} < 0$$

for all $p_{a1} > \frac{1}{2}\delta p$. This means that all bubbles of radius $R_1 < 4\sigma/3\delta p$, with $\delta p > 0$, are dynamically stable. However, these bubbles soon dissolve through diffusion of air out of the bubble (Plesset & Epstein 1950).

For the case $\delta p \leqslant 0$, with $p_{a0} > |\delta p|$, the equilibrium radius is

$$R_0 = \frac{2\sigma}{\delta p + p_{a0}},$$

and (9) becomes
$$\left[\frac{df(R)}{dR}\right]_{R=R_0} = \frac{\delta p + p_{a0}}{2\sigma}(\delta p - 2p_{a0}).$$

Since $p_{a0} > |\delta p|$, one obtains $\left[\frac{df(R)}{dR}\right]_{R=R_0} < 0.$

This means that if the vapour pressure, p_v, is less than the atmospheric pressure, p_∞, then any air bubble existing in the liquid of radius R_0, with $R_0 = 0$, is dynamically stable. However, these bubbles also slowly dissolve through diffusion of air out of the bubble. Evidence for the existence of these bubbles can be found in observing water as it is slowly heated. Near the temperature of 80 °C, where the vapour pressure, p_v, is still less than the atmospheric pressure, p_∞, corresponding to the condition $\delta p < 0$, these bubbles can be seen floating within the body of the liquid or clinging to the container walls. Their duration, although limited by diffusion, is still long enough for visual observation.

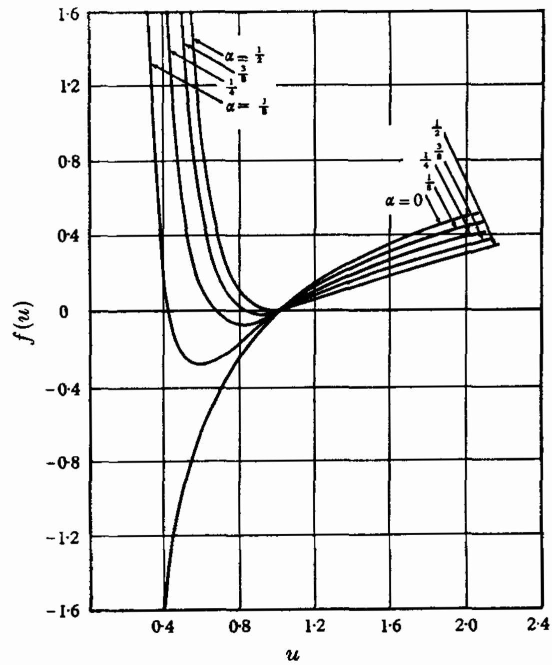

FIGURE 3. The function $f(u)$, the dimensionless form for the total pressure $f(R)$ acting on the bubble wall (see (10)), for various values of α. The case $\alpha = 0$ corresponds to a vapour bubble.

A convenient way to plot $f(R)$ as a function of R for various initial values of p_{a0} is to write $p_{a0} = \alpha\delta p$ and then change $f(R)$ to dimensionless form by dividing through by $\delta p + p_{a0}$. Thus

$$\frac{f(R)}{\delta p + p_{a0}} = \frac{\delta p}{\delta p + p_{a0}} + \frac{p_{a0}}{\delta p + p_{a0}} \frac{R_0^3}{R^3} - \frac{2\sigma}{(\delta p + p_{a0})\, R};$$

and, with $u = R/R_0$ and
$$(1+\alpha)\frac{f(R)}{\delta p + p_{a0}} = f(u),$$

one gets
$$f(u) = 1 - \frac{1+\alpha}{u} + \frac{\alpha}{u^3}. \tag{10}$$

Figure 3 shows $f(u)$ as a function of u for various values of α. The case $\alpha = 0$ corresponds to the growth of a vapour bubble.

The preceding analysis has shown that for the growth of a bubble as governed by the equation of motion

$$R\ddot{R} + \tfrac{3}{2}\dot{R}^2 = \frac{\delta p + p_{a0}(R_0/R)^3 - (2\sigma/R)}{\rho}, \tag{11}$$

with initial conditions R_0 and $\dot{R}_0$, the range of equilibrium bubble radii which are dynamically unstable is

$$\frac{4\sigma}{3\delta p} \leqslant R_0 \leqslant \frac{2\sigma}{\delta p},$$

where

$$\tfrac{1}{2}\delta p \geqslant p_{a0} \geqslant 0.$$

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